Problem set for Platforms for Quantum Technologies
Prof. Dr. H. Bluhm, II. Physikalisches Institut, RWTH Aachen
Dr. A. Sharma, II. Physikalisches Institut, RWTH Aachen

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Contact person: Dr. A. Sharma
sharma@physik.rwth-aachen.de

Definitions:

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\mathrm{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \mathrm{R}_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}, |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Problem 1: Bloch sphere and qubit dynamics

1. Let us consider the Hamiltonian, $H = \hbar\omega\sigma_X$, which can be regarded as the tunnel-only coupled charge qubit Hamiltonian. Find the solution of the time-dependent Schrödinger equation, $i\hbar\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$ i.e. $|\Psi(t)\rangle = e^{-iHt/\hbar}|\Psi(t=0)\rangle$ with the initial condition $|\Psi(t=0)\rangle = |0\rangle$. Find the probability $|\langle\Psi(t=0)|\Psi(t)\rangle|^2$.
Hint: Use the relation, $e^{i\alpha(\boldsymbol{n}\cdot\boldsymbol{\sigma})} = \sigma_0 cos(\alpha) + i(n_X\sigma_X + n_Y\sigma_Y + n_Z\sigma_Z) sin(\alpha)$.

2. Show that the unitary operator,
$$R_z\left(\frac{\pi}{2} + \phi_2\right) H R_z\left(\theta_2 - \theta_1\right) H R_z\left(-\frac{\pi}{2} - \phi_1\right),$$
moves the quantum state parametrized on the Bloch sphere by the angles (θ_1, ϕ_1) into the quantum state (θ_2, ϕ_2). With $(\theta_1, \phi_1) \equiv (0, \pi)$ and $(\theta_2, \phi_2) \equiv (\pi, 0)$ plot and visualize each step on the Bloch sphere.
Hint: write $HR_z(\alpha)H$ *in terms of identity and Pauli matrices.*