Problem set on Platforms for Quantum Technologies
Prof. Dr. H. Bluhm, II. Physikalisches Institut, RWTH Aachen
Dr. A. Sharma, II. Physikalisches Institut, RWTH Aachen

M3 Exercises, 23.03.2020, Session II - 16:15 - 17:00

Contact person: Dr. A. Sharma
sharma@physik.rwth-aachen.de

Problem 1:   Charge qubit

In the lecture, it was shown how a double-dot potential can be defined to create a charge qubit. In this exercise, the focus will be to make rough estimates of the relevant energy scales and the energy diagram.

1. The conduction band minimum of a GaAs/AlGaAs structure was discussed in the lecture. The 2DEG is formed in a triangular quantum well, the eigenenergies of which can be calculated numerically. For simplicity, let us assume a square quantum well with a thickness of $d_z = 10\,\text{nm}$ to host the 2DEG. Calculate the energy difference between the edge of the first and the second energy band of this quantum well, where each energy band corresponds to a different z-eigenstate. *Hint: The 2DEG eigenenergies are given by $E_n(k_{x,y}) = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2) + \frac{\hbar^2\pi^2}{2m^*}\frac{n^2}{d_z^2}$ where the first and second term are the continuous and quantized energy, respectively. Here, k_x and k_y are the momenta along $x-$ and $y-$ directions, respectively, n is the band index, $\hbar = 6.58 \times 10^{-16}\ eV.s$, and $m^*(GaAs) = 0.07\ m_e$ with $m_e = 0.51\ MeV/c^2$ and c is speed of light.*

2. Now, let us turn on negative voltages on some of the metallic gates to form a single dot with a parabolic potential in the xy-plane (in the plane of the 2DEG),

$$V(x,y) = a(x^2 + y^2).$$

   A representative potential for the electrons in the 2DEG plane is shown in the figure below where the metallic gates are indicated in white. Roughly estimate the curvature in the vicinity (about 30 nm) of the potential minimum from the 1D plot (left in Fig. 1). Then calculate the single electron energy, i.e. the energy required to excite the electron to the next orbital in this confinement potential. Sketch the shape of the electron wave function of the ground state and give an estimate of its

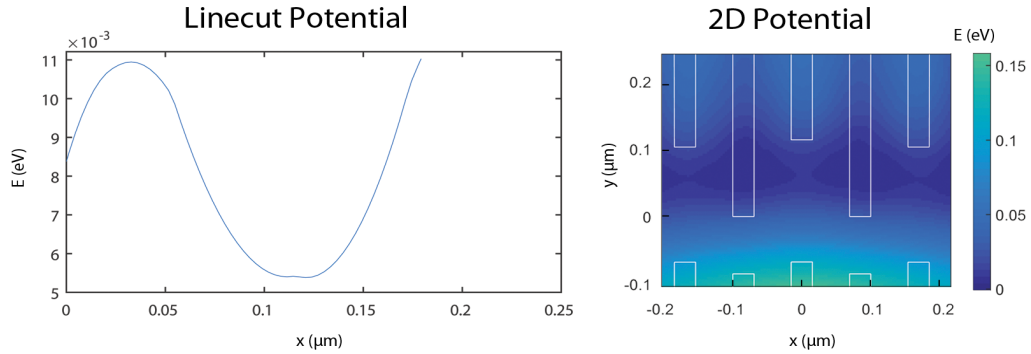width Δx. *Hint: use the well known results/ formulas of the harmonic oscillator.*

Fig. 1: Left: Potential for the electrons in 2DEG along the $x-$ direction, right: schematic of the device, with the metallic gates being depicted in white, showing the 2D potential map across the $x - y$ plane.

3. What is the charging energy required to charge the dot with a second electron? *Hint: calculate the capacitance using $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1} and $\varepsilon_r(GaAs) = 13.1$, and assume the dot to be a homogeneously charged sphere with reasonable radius. The value of electron charge, e $= 1.6 \times 10^{-19}$ C.*

4. A second dot is formed at a distance d with respect to the first dot. Estimate the distance d from the 2D plot (right in Fig. 1) and calculate the inter-dot coupling energy, i.e. changes in the Coulomb energy required to fill the second dot depending on whether an electron is present in the first dot.

5. In the lecture, the energy diagram of a charge qubit as a function of the detuning, ϵ, was plotted. Let us consider, the eigenstates $|\psi_i\rangle$ $(i = \{1, 2\})$ to be a function of the detuning as well. On assuming $H = \frac{\epsilon}{2}\sigma_z + \frac{\Delta}{2}\sigma_x$, show that the local slope $\frac{dE_i}{d\epsilon}$ (at a given ϵ) indicates the amount of $|L\rangle$ component in the eigenstate $|\psi_i\rangle$ i.e. $\frac{dE_i}{d\epsilon} \propto \langle\psi_i|L\rangle$. *Hint: using pertubation theory show that,*

$$\frac{dE_i}{d\epsilon} = \frac{\langle\psi_i|\,\sigma_z\,|\psi_i\rangle}{2},$$

and relate the right hand side of the above equation to the $|L\rangle$ component: $|\langle\psi_i|L\rangle|^2$.