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PLATFORMS FOR QUANTUM TECHNOLOGIES – QUANTUM SIMULATORS
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EXERCISE 1

I. QUESTION 1

Explain the principles of laser cooling mechanisms and the conditions for cooling. How is laser cooling implemented in experiments and which role does it play in the quest for achieving quantum gases? Why is it very difficult to reach quantum gases by Doppler laser coolingd directly? Discuss the magneto-optical trap and, in particular, comment on the laser and magnetic field configuration, the required atomic level structure, and the selection rules involved. Make a sketch of the relevant levels and transitions to illustrate your verbal explanation.

II. QUESTION 2

We consider a particle in an optical lattice with a potential

$$V(x) = V_0 \sin^2(kx), \tag{1}$$

the period of which we denote by a.

1. Sketch and explain the energy spectrum of a particle in such a potential (2 points). State the (approximate) energies $E_{q=0}^{(m)}$ of the quasi-momentum $q = 0$ states for the different Bloch bands labeled by the index m in the limit $V_0 \to 0$. (1 point)
2. State the Bloch theorem and explain how a general wave function of the atoms in such a periodic potential looks like (2 points).
3. We label the Bloch band by an index m starting from $m = 0$ for the lowest band. The approximate Bloch wave functions in a very weak optical lattice are given by $\psi_{q=0}^{m}(x) \simeq \cos(mkx)$ for even m and $\psi_{q=0}^{m}(x) \simeq \sin[(m+1)kx]$ for odd m. Check if this wave function fulfils the Bloch theorem (1 point).
4. A time-periodic lattice modulation with small amplitude ϵ creates a perturbative potential $\delta V(x)\sin(\omega t) = \epsilon V_0 \sin^2(kx)\sin(\omega t)$ added to $V(x)$. We plan to use the modulation to drive transitions from the ground state of the lattice to excited states. Discuss the transition matrix element $\langle \psi_{q=0}^{m=0} | \delta V(x) | \psi_{q=0}^{m'} \rangle$ and explain, which transitions $m = 0 \to m'$ are possible and which ones are not (4 points).
5. Consider the case that a constant external force F is applied to the lattice, which accelerates the atoms according to $F = \hbar \frac{dq}{dt}$. Integrate the equation of motion (1 point), and describe and sketch the dynamics $q(t)$ of the atom in the band structure, starting from $q(0) = 0$ (3 points).
6. In the limit of a sufficiently strong lattice lattice potential V_0, we can approximate the dispersion relation of the lattice in the lowest band by $E(q) = -2t\cos(qa)$. Compute the velocity of the atom $v(t)$ subject to a constant force F using $v(q) = \frac{1}{\hbar}\frac{dE(q)}{dq}$ (1 point) and sketch (1 point). Comment on the difference between the two quantities $\hbar q(t)$ and $mv(t)$ (2 points). Calculate the dynamics of the wave packet in position space (2 points).

Hints:
$\sin(x \pm y) = \sin x \ \cos y \pm \cos x \ \sin y$
$\cos(x \pm y) = \cos x \ \cos y \mp \sin x \ \sin y.$